Coherent Pricing of Life Settlements Under Asymmetric Information

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Abstract

Although life settlements are advertised to deliver a profitable investment opportunity with a low correlation to market systematic risk, recent investigations reveal a discrepancy of expected and realized returns. While thus far this discrepancy has been attributed to the (allegedly) poor quality of the underlying life expectancy estimates, we present a different explanation of the seemingly high reported expected returns based on adverse selection. In particular, we provide a coherent pricing mechanism and pricing formulas in the presence of asymmetric information with respect to the underlying life expectancies. Therefore, our study sheds light on the nature of the “unique risks” within life settlements as recently discussed in the financial press.

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1 Introduction

While markets for mortality-linked securities are only slowly emerging in recent years, one segment has established itself as an abiding investment opportunity: The life settlement market. Evolving from so-called “viatical settlements” popular in the late 1980s that only targeted severely ill life insurance policyholders, life settlements generally involve senior insureds with below average life expectancies. Within such a transaction, both the liability of future contingent premiums and the benefits of a life insurance contract are transferred from the policyholder to a life settlement company, which may further securitize a bundle of contracts in the capital market (cf. Chen et al. (2011), Stone and Zissu (2006)). However, typical investments only involve a limited number of contracts from rather specific population strata. Thus, in contrast to other mortality derivatives, the key risk factors for a life settlement are idiosyncratic in nature, i.e. they derive from uncertainties in the individual rather than population mortality trends. Nevertheless, a sizable life settlement market may help promote other mortality-linked capital markets by increasing investors’ familiarity and awareness. In particular, all these securities are jointly advertised to deliver profitable investment opportunities with a low correlation to market systematic risk (cf. Cowley and Cummins (2005)).

However, at least for the life settlement market, recent investigations reveal a discrepancy of expected and realized returns (see Beyerle (2007) or Gatzert (2010)). More specifically, while expected returns calculated on a policy-by-policy basis range from 8-12% annually (cf. Gatzert (2010)), according to Braun et al. (2012) open-end life settlement funds between 2004 and 2010 on average returned approximately 4.8%, which considering substantial lock-up periods and redemption fees only slightly exceeds risk-free rates. While thus far this gap between anticipated and realized returns has been attributed to the bad quality of the underlying life expectancy estimates (see Gatzert (2010) and references therein), potential systematic biases should have been swiftly corrected whereas unsystematically erroneous estimates by themselves do not provide a coherent explanation for an aggregate underperformance. It is exactly this difficulty in assessing the expected return of a life settlement due to “unique risks” that led rating agencies to decline rat-
ing these “death-bet securities”.

This, on the other hand, could impede the further development of this market, as ratings are usually essential to a broad investor interest.

In this paper, we propose a different viewpoint on the seemingly high reported excess returns based on adverse selection. We start by presenting a simple one-period expected utility model to derive the offer price in a competitive life settlement market. Specifically, by analyzing a representative policyholder’s decision to settle her policy, we derive the per-policy expected profit for a representative life settlement company in two cases: (1) When the policyholder’s lifetime distribution is public information, i.e. if the life settlement company and the policyholder have the same (symmetric) information on the policyholder’s condition; and (2) when the policyholder has private insights that improve her assessment of her lifetime distribution, i.e. when there exists hidden (asymmetric) information. In the former case (1), the life settlement company relies on its best estimate of the policyholder’s expected lifetime to derive the actuarially fair offer price. In contrast, if information is asymmetric, a rational life settlement company will not directly use the (unconditional) expected lifetime for pricing, even if the estimation itself is unbiased and the company is risk-neutral. Rather, it will adjust the pricing scheme to cover possible one-sided losses because profitable policyholders may walk away from the transaction whereas unprofitable offers are accepted. In particular, this renders the offer price lower than the actuarially fair price – which is typically the benchmark used in practice.

Building on these insights, we then derive applicable pricing formulas for life settlement transactions within a lifetime utility framework. More precisely, we rely on the frailty model from Vaupel et al. (1979) and corresponding population parameter estimates from Manton et al. (1986) to introduce heterogeneity with respect to individuals’ mortality rates. By evaluating the policyholder’s (optimal) lifetime utility of consumption and bequest based on the private information regarding her lifetime distribution in a simple model setup, we then derive the threshold set for accepting the settlement offer – and, thus, the offer price. Moreover, we discuss possible generalizations, particularly when the policyholder has the option to settle at various dates. Here, the

\(^1\text{Cf. Wall Street Journal, 04/22/2011. “AIG Tries to Sell Death-Bet Securities” (by Leslie Scism). See also Standard and Poor’s (2011).}\)
derivation of the offer price requires the solution of an optimal stopping problem and uncertainties in the population mortality trend become material.

Our numerical example calculations and sensitivity tests reveal that the impact of asymmetric information on the offer price varies, depending on various parameter choices. If policyholders receive little utility from their life insurance contracts and the settlement is highly priced, the influence of private information on pricing can be small. However, if the policyholder is on the verge of keeping her policy, if pricing is not beneficial, or if she can settle at various dates against a small cost, the information asymmetry may considerably affect the equilibrium offer price. In particular, the discrepancy of “expected” and realized returns in our example calculations can be as high as 5.72%, which roughly falls in the range of the deviations of “expected” and realized returns in the life settlement market reported in the literature.

Related Literature and Organization of the Paper

Adverse selection in insurance markets of course is a very important topic and has triggered a large number of contributions in the economics and insurance literature. We refer the interested reader to recent surveys by Chiappori and Salanie (2013), Dionne et al. (2013), and references therein for details. One of the most important predictions under asymmetric information is a positive relationship between risk and coverage, so that basing the price on the population-weighted actuarially fair rate will lead to losses for the seller of insurance. For instance, in the market for life annuities, individuals with a high probability of survival – i.e. those posing a high risk for the annuity provider – will be more inclined to purchase coverage whereas those with a high probability of death (low risks) find the company’s offer less attractive. Therefore, firms must adjust upward the estimate of average survival probability for the pool of annuitants relative to the general population when pricing the annuities (see e.g. Finkelstein and Poterba (2002, 2004); see also Sheshinski (2007) for implications). Failure to account for this will lead to losses for the provider – as e.g. documented by Rothschild (2009) based on evidence from the British Life Annuity Act of 1808.

The mechanism in the current paper is closely related but reversed: A policyholder will be more
inclined to settle (i.e. sell) her policy if she is a low risk, i.e. if she has a low mortality rate. If life settlement companies do not take this into account then they will incur losses because they offer a too high price for the contracts. The same effect was already pointed out by Polborn et al. (2006) in a three period model with uncertainty about the risk type and the insurance demand type that is resolved after the first period. In particular, they show that low risk types are more inclined to “sell off” their policies and, by relying on corresponding results from Villeneuve (2003) and Hoy and Polborn (2000), that the appropriate selling price in a competitive equilibrium will thus be lower than that based on the population average. Moreover, the mechanism echoes results from the literature on commitment – or rather the lack thereof – in long term insurance contracts: Low risks drop out leaving the insurer with a pool of adversely selected risks (see e.g. Hendel and Lizzeri (2003) for corresponding empirical results, and particularly the role of front-loading, in the case of life insurance).

Thus, our primary contribution lies in the adaptation of these ideas to the case of life settlements and their financial analysis. In particular, after establishing the basic mechanism in a simple one-period model, we develop pricing formulas in a multi-period life-cycle model with heterogeneous life expectancies. The insights based on our numerical results supplement a missing piece to the puzzling discrepancy between expected and realized returns observed in this market. These findings should help to promote the life settlement market, and thereby the mortality-linked capital market as a whole.

The remainder of this paper is organized as follows: The simple one-period model is introduced in Section 2. The generalized lifetime utility framework to derive the offer price as well as extensions are presented in Section 3. Section 4 contains our numerical analyses, and finally Section 5 concludes.
2 One-Period Model for Life Settlements

In order to obtain our key implication while keeping the setup as succinct as possible, we commence by looking at a simple one-period expected utility model. More precisely, we consider a representative policyholder endowed with a one-period term-life insurance policy with face value $F$, no future contingent premiums, and zero cash surrender value. We identify the policyholder’s condition by her survival probability to the end of the period, $p$. Moreover, we denote by $u(x)$ the policyholder’s utility function when surviving until the end of the period and receiving $x$ as a payment related to her life insurance policy, whereas $v(y)$ denotes the policyholder’s bequest function when she dies within the period and receives $y$ from the policy.\(^2\) Assume both $u(\cdot)$ and $v(\cdot)$ satisfy common conditions for utility functions such as increasingness and concavity. Finally, assume that the life insurance policy is nonseparable, i.e. the policyholder cannot partially settle her policy.\(^3\)

Without a secondary life insurance market, the policyholder has no other choice but to simply retain her policy. Her expected utility, $U^r$, can then be written as

$$U^r = p \times u(0) + (1 - p) \times v(F).$$  \hspace{1cm} (1)

If there exists a secondary life market, on the other hand, the policyholder will have the additional option of settling the policy with a representative life settlement company. Here we assume the secondary life market is competitive and the life settlement company is risk-neutral. In the following two subsections, we derive the equilibrium offer price for a life settlement transaction with symmetric and asymmetric information on the policyholder’s condition $p$, respectively.

\(^2\)Note that here the policyholder’s wealth levels in different states are implicitly incorporated in $u(x)$ and $v(y)$. Therefore, here $u(x)$ and $v(y)$ can be simply interpreted as “utilities from life insurance benefits”, where of course $x = 0$ if the policy is not settled.

\(^3\)While this assumption is in line with practice since actual transactions typically entail “entire policies”, partial settlement offers the interesting possibility of serving as a screening mechanism to reveal private information. However, a detailed analysis will require a careful consideration of the various issues associated with screening in this case such as the exclusivity of the contractual relationship or transferability of the revealed information. Since we are primarily interested in pricing actual transactions, a detailed exploration of this possibility goes beyond the scope of this paper and we leave it for future research.
2.1 Symmetric Information

First, assume that $p$ is publicly observable by both the policyholder and the life settlement company. Then the expected profit of the life settlement company when offering a price $x$ to the policyholder can be written as

$$\pi = (1 - p) \times F - x(1 + R_f),$$

where $R_f$ is the risk-free interest rate. Since a competitive equilibrium will yield zero expected profits, the equilibrium offer price, $OP^{sym}(p)$, will trivially be:

$$OP^{sym}(p) = \frac{(1 - p) \times F}{1 + R_f}.$$

Thus, the policyholder’s expected utility when settling, $U^s$, can be written as

$$U^s = p \times u((1 - p)F) + (1 - p) \times v((1 - p)F). \quad (2)$$

The policyholder will choose to settle her policy if and only if $U^s \geq U^r$. Note that this condition is automatically valid when $u(\cdot) = v(\cdot)$, but it will not always be met under more general utility and bequest assumptions.

2.2 Asymmetric Information

Now consider the case in which the information on $p$ is asymmetric. More specifically, we assume that $p$ is only observable by the policyholder, whereas the life settlement company has to turn to an expert third party (a so-called life expectancy provider) to compile an estimate denoted by $\bar{p}$. Let $f(p|\bar{p})$ denote the probability density of $p$ given $\bar{p}$. In order to price the transaction, the life settlement company now has to rely on information from $\bar{p}$, or more precisely, $f(p|\bar{p})$.

We first consider the case in which the offer price is solely determined from $f(p|\bar{p})$ without taking into account the policyholder’s behavior. The associated solution, denoted as the actuarially
fair price, \( OP^a(\bar{p}) \), is then a simple extension of \( OP^{sym}(p) \) as defined in Section 2.1:

\[
OP^a(\bar{p}) = \int_0^1 (1 - p)f(p|\bar{p}) dp \times \frac{F}{1 + R_f} = \frac{\mathbb{E}[(1 - p)|\bar{p}] \times F}{1 + R_f},
\]

which simply results in \( OP^a(\bar{p}) = \frac{(1 - \bar{p}) \times F}{1 + R_f} \) if the estimate \( \bar{p} \) is unbiased. While \( OP^a(\bar{p}) \) is the benchmark used in practice,\(^4\) in what follows we show that \( OP^a(\bar{p}) \) is not economically correct in the sense that it does not consider the policyholder’s decision process. Instead, it comes with the strong – and possibly untenable – assumption that policyholders will always choose to settle.

The expected utility for a policyholder with condition \( p \) when retaining the policy, \( U^r(p) \), is given by Equation (1). Her expected utility when settling, \( U^s(p, OP) \), for any exogenously given offer price \( OP \), on the other hand, can be written analogously to Equation (2) as

\[
U^s(p, OP) = p \times u(OP \times (1 + R_f)) + (1 - p) \times v(OP \times (1 + R_f)).
\]

Therefore, given \( OP \), a rational policyholder will settle her term-life policy if and only if

\[
0 \leq U^s(p, OP) - U^r(p) = p \times u(OP \times (1 + R_f)) + (1 - p) \times v(OP \times (1 + R_f)) - p \times u(0) - (1 - p) \times v(F)
\]

\[
= p \times \left( u(OP \times (1 + R_f)) - u(0) + v(F) - v(OP \times (1 + R_f)) \right) + v(OP \times (1 + R_f)) - v(F).
\]

Since \( OP \) is naturally bounded between 0 and \( \frac{F}{1 + R_f} \), it is easy to verify that \( u(OP \times (1 + R_f)) - u(0) + v(F) - v(OP \times (1 + R_f)) > 0 \), and \( v(OP \times (1 + R_f)) - v(F) \leq 0, \forall OP \). Hence, the above condition for settling can be equivalently written as

\[
p \geq \frac{v(F) - v(OP \times (1 + R_f))}{u(OP \times (1 + R_f)) - u(0) + v(F) - v(OP \times (1 + R_f))} \triangleq p^*(OP) \ (\in [0, 1]).
\]  

With Equation (4), the competitive equilibrium offer price under rational expectations, \( OP^e \), can

\(^4\) See e.g. Deloitte (2005) or www.lifesettlementguide.org.
then be determined from the following equation:

\[ OP^e(\bar{p}) \triangleq \arg\max_x \left\{ \int_{p^*(x)}^1 p \cdot f(p|\bar{p}, p \geq p^*(x)) dp = 0 \right\}, \]  \hspace{1cm} (5)

where the integration on the right-hand side calculates the expected profit of the life settlement company for given offer price \( x \) by only (rationally) taking into account conditions \( p \) above the threshold \( p^*(x) \) as defined in Equation (4). The equilibrium offer price, \( OP^e(\bar{p}) \), is then the highest price that maintains a zero expected profit for the company.

The following proposition now provides a comparison result that gives rise to an alternative explanation for the discrepancy between expected and realized returns in the life settlement market.

**Proposition 2.1.** In the presence of asymmetric information with respect to \( p \), the rational expectation offer price, \( OP^e(\bar{p}) \), will be smaller than \( OP^a(\bar{p}) \) for all estimates \( \bar{p} \).

A proof is presented in Appendix A. The key insight here is that a policyholder looking to settle her policy will reject the offer when the value of the policy is far underestimated. On the other hand, she will be happy to settle the policy for the offered price if it considerably exceeds the intrinsic value. Since the life settlement company cannot observe the “true” condition \( p \), it is not able to determine whether the offer price is too low or too high before the policyholder makes her choice. Therefore, as a way to balance expected profits in the competitive equilibrium, the life settlement company has to shift its pricing schedule to cover the possible tail loss. As a consequence, if the life settlement company determines its offer price based on Equation (3) and a given rate \( R \) set according to investors’ return expectations, the resulting returns will – on average – be lower than \( R \). Ascertaining a genuine return of \( R \) requires its incorporation into Equation (5) for \( OP^e(\bar{p}) \), where the decision to settle is taken into account. Thus, if life settlement companies or funds rely on Equation (3) for pricing their settlements, discrepancies between “expected” and realized returns arise naturally in the presence of asymmetric information within the life settlement market.

Finally, it is worth pointing out that Equation (5) may not always have a solution. For instance,
if the policyholder is risk-neutral, we have
\[ \int_{p^*(x)}^{1} \left( (1 - p)F - x(1 + R_f) \right) f(p|\bar{p}, p \geq p^*(x)) \, dp < 0, \forall x > 0, \]
so that the only admissible offer price is \( x = 0 \) for which clearly \( p^*(0) = 1 \). This is not surprising, since we are in the situation of a so-called “Lemon Market” as described by Akerlof (1970) in his seminal contribution. But even in the case of risk-averse policyholders, the market may break down, for instance, if \( R_f \) is sufficiently high or if there is a significant transaction cost associated with eliciting the life expectancy estimation.

3 The Extended Framework

In Section 2, we derived equilibrium offer prices for life settlement transactions under asymmetric information with respect to the policyholder’s lifetime distribution in a simple one-period model. Building on the gained insights, in this section, we extend the simple setup to arrive at more coherent pricing formulas in a more realistic environment. Specifically, we consider a whole-life insurance policy with level annual premium \( P \) and death benefit \( F \). We start by introducing heterogeneity in the expected lifetime distribution via a frailty model for generation life tables. Subsequently, we describe how to evaluate the policyholder’s decision process in a lifetime utility framework and derive corresponding pricing formulas. Finally, we discuss possible generalizations.

3.1 Heterogeneous Generation Life Tables

Heterogeneity with respect to individual mortality rates has been long studied in the demographic as well as in the life insurance literature (see e.g. Vaupel et al. (1979) or Hoermann and Russ (2008)). A common approach to capture individual heterogeneity are so-called frailty models, i.e. to use a frailty factor by which the actual individual survival probabilities deviate from population probabilities.
Here, we rely on a popular choice, namely the Gamma frailty model (see Vaupel et al. (1979), Manton et al. (1986), and references therein). Define \( \tau p_x(T) \) as the \( \tau \)-year survival probability for some cohort of \( x \)-year olds at time \( T \), and \( \tau p^j_x(T) \) as the corresponding \( \tau \)-year survival probability for individual \( j \). Define the individual’s time-\( T \) forward force of mortality as

\[
\mu^j_T(\tau, x) = -\frac{\partial}{\partial \tau} \log \{ \tau p^j_x(T) \},
\]

and assume that \( \mu^j_T(\tau, x) \) satisfies

\[
\mu^j_T(\tau, x) = z^j \times \mu_T(\tau, x).
\]

Here, \( \mu_T(\tau, x) \) is the population-level forward force of mortality and \( z^j \) is the individual frailty factor, which is assumed to follow a Gamma distribution, i.e. the probability density is given by

\[
f(z) = \frac{(z/\zeta \gamma^2)^{1/\gamma^2} \exp(-z/\zeta \gamma^2)}{z \Gamma(1/\gamma^2)}
\]  

(6)

with mean frailty \( \zeta \) and coefficient of variation \( \gamma \) (cf. Manton et al. (1986)). It is easy to verify that under these assumptions, we have:

\[
\tau p^j_x(T) = (\tau p_x(T))^{z^j}.
\]  

(7)

Therefore, for given parameters \( \zeta, \gamma \), and cohort generation life table \( \{ \tau p_x(T) \}_{\tau \geq 0} \), from Equations (6) and (7), we can derive individual-level heterogeneous life tables \( \{ \tau p^j_x(T) \}_{\tau \geq 0} \) \( j=1,2,\ldots \).

### 3.2 Evaluation of the Policyholder’s Decision

Similarly to Section 2.2, for a given offer price, the policyholder’s rational behavior is characterized by a threshold set, which may be derived from a lifetime utility model (see e.g. Chai et al. (2011) and references therein for more details on lifetime utility models). For simplicity, we assume that
the policyholder can only settle her policy at time $T$. Extensions are discussed in Section 3.4.

Within the typical lifetime utility framework with time-separable preferences, the value function for the $x$-year old policyholder $j$ with initial wealth $W_0$ and life table $\{\tau p_x^j(T)\}_{\tau \geq 0}$, when retaining the policy at time $T$, $V_r^T(W_0, j)$, is defined as

$$V_r^T(W_0, j) = \max_{c_T} \sum_{\tau=0}^{\omega-x-1} \tau p_x^j(T) \beta^\tau u(c_\tau - P) + \sum_{\tau=0}^{\omega-x-1} (\tau p_x^j(T) - \tau+1 p_x^j(T)) \beta^{\tau+1} v(W_{\tau+1} + F), \quad (8)$$

s.t.

$$W_\tau = (W_{\tau-1} - c_{\tau-1}) \times \frac{1}{p(\tau - 1, 1)}, \quad \tau = 1, \ldots, \omega - x.$$

Here $F$ is the death benefit, $P$ is the periodic contingent premium, $\beta$ is the policyholder’s time discount factor, $p(t, \tau)$ is the time $t$ price of a zero coupon bond with maturity $t + \tau$, $u(\cdot)$ is the utility function of period consumption $c_\tau$ net of premium payment, and $v(\cdot)$ is the bequest function.$^6$

The value function when settling her policy at offer price $OP$, $V_s^T(W_0, OP, j)$, on the other hand, is analogously defined as

$$V_s^T(W_0, OP, j) = \max_{c_T} \sum_{\tau=0}^{\omega-x-1} \tau p_x^j(T) \beta^\tau u(c_\tau) + \sum_{\tau=0}^{\omega-x-1} (\tau p_x^j(T) - \tau+1 p_x^j(T)) \beta^{\tau+1} v(W_{\tau+1}), \quad (9)$$

s.t.

$$W_1 = (W_0 - c_0 + OP) \times \frac{1}{p(0, 1)};$$

and

$$W_\tau = (W_{\tau-1} - c_{\tau-1}) \times \frac{1}{p(\tau - 1, 1)}, \quad \tau = 2, \ldots, \omega - x.$$

Therefore, for each given offer price $OP$, policyholder $j$ makes her decision by comparing the

$^5$Here $\omega$ denotes the limiting age, i.e. $\tau p_x^j(T) = 0, \forall \tau \geq \omega - x$.

$^6$Note that for simplicity and to focus on the effects of adverse selection on settling the policy, we assume that the utility functions are independent of the health state $z_j$. Of course, our model framework can be easily extended to health-state dependent utility functions. In particular, the decision to settle may then be influenced by how the health state affects individuals’ (marginal) utilities – which is the subject of an ongoing debate in the economics literature (see e.g. Viscusi and Evans (1990), Evans and Viscusi (1991), or Finkelstein et al. (2013)).
value functions and will only choose to settle if \( V_s^r(W_0, OP, j) \geq V_r^r(W_0, j) \). Note that both \( V_s^r(W_0, j) \) and \( V_r^r(W_0, OP, j) \) depend on the individual life table \( \{\tau p_x^j(T)\}_{\tau \geq 0} \), or more precisely, the frailty factor \( z^j \). Define

\[
\Omega(\text{OP}) = \{ z^j : V_s^r(W_0, OP, j) \geq V_r^r(W_0, j) \}.
\]

Thus, for a given offer price \( \text{OP} \), \( \Omega(\text{OP}) \) is the threshold set of \( z^j \) in which settling is preferred to retaining.

### 3.3 Coherent Pricing Formulas

If there is no private information on the policyholder’s expected lifetime distribution, in analogy to Section 2.1, the offer price will be calculated following the actuarial equivalence principle. More precisely, for the \( x \)-year old individual \( j \) with publicly observed future survival probabilities \( \{\tau p_x^j(T)\}_{\tau \geq 0} \) and a whole-life policy with face value \( F \) and premium \( P \) at time \( T \), the equilibrium offer price, \( OP^{\text{sym}}(j) \), can be written as

\[
OP^{\text{sym}}(j) = \sum_{\tau=1}^{\omega-x} \left[ \left( \tau p_x^j(T) - \tau p_x^j(T) \right) \times \frac{F}{(1+R)^\tau} - \tau p_x^j(T) \times \frac{P}{(1+R)^{\tau-1}} \right],
\]

where \( R \) is the hurdle rate set by the life settlement company according to investors’ return expectations. A hurdle rate in excess of the risk-free rate \( R^f \) generalizes the considerations from the previous section and may arise due to, e.g., capital market imperfections as in Froot (2007).

With asymmetric information introduced in the form of \( z^j \), on the other hand, akin to the ideas presented in Section 2.2, a coherent pricing formula needs to explicitly take into account the policyholder’s decision process. Specifically, denote by \( \bar{z} \) the estimate of \( z^j \) supplied by the life expectancy provider, and by \( f(z^j | \bar{z}) \) the corresponding probability density of \( z^j \) given \( \bar{z} \). Note that we assume that \( \bar{z} \) includes all the available information on the individual including possible impairments. The offer price for the \( x \)-year old individual with policy face value \( F \) and estimate \( \bar{z} \)
at time $T$, $OP_c(z)$, can then be derived via the following equation:

$$
OP_c(z) \triangleq \arg\max_y \left\{ \int_{\Omega(y)} \left( \sum_{\tau=1}^{\omega-x} \left[ (\tau-1)p_x^j(T) - \tau p_x^j(T) \right] \times \frac{F}{(1 + R)^\tau} - \tau p_x^j(T) \times \frac{P}{(1 + R)^{\tau-1}} - y \right) f \left( z^j | \tilde{z}, z^j \in \Omega(y) \right) dz^j = 0 \right\} \quad (11)
$$

Similarly to Section 2.2, the integration on the right-hand side calculates the expected profit of the life settlement company when offering $y$ for the whole-life policy with the hurdle rate set at $R$.

Also similarly to Section 2.2, we define the actuarially fair offer price given $\tilde{z}$ as $OP_a(\tilde{z}) = \mathbb{E}[OP_{sym}(j) | \tilde{z}]$. The following proposition extends Proposition 2.1 to the multi-period framework. A proof is presented in Appendix B.

**Proposition 3.1.** Assume $\Omega(OP) = [0, z^*(OP)]$. In the presence of asymmetric information with respect to $z^j$, the rational expectation offer price, $OP_c(z)$, will be smaller than $OP_a(z)$ for all estimates $\tilde{z}$.

### 3.4 Generalization

In Section 3.2, we require that the policyholder can only settle her policy at time $T$. However, this restriction may be problematic since the policyholder might also have the option to settle in future periods. If we allow the policyholder to settle in any period, the value function when settling, $V_T^*(W_0, OP(z_T), j)$ remains essentially unchanged, and depends on the life expectancy estimate at time $T$, $z_T$. The value function when retaining at time $T$, $\hat{V}_T(W_0, j)$, on the other hand, will be modified, since it now includes the possibility to settle in future periods.

More specifically, the policyholder now solves a dynamic program:

$$
\hat{V}_T(W_0, j) = \max_{c_0} u(c_0 - P) + \beta \times \mathbb{E} \left[ (1 - p_x^T(T)) \times v(W_1 + F) + p_x^T(T) \times \hat{V}_{T+1}(W_1, j) \right],
$$

(12)

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7This assumption is verified in our numerical analysis for the parameters under consideration (cf. Figure 3(b)). However, providing a general validation is difficult without imposing further conditions.
Coherent Pricing of Life Settlements

\[ W_1 = (W_0 - c_0) \times \frac{1}{p(0,1)}, \]

and

\[ \hat{V}_{T+1}(W_1, j) = \max \left( V^r_{T+1}(W_1, j), V^s_{T+1}(W_1, OP(\bar{z}_{T+1}), j) \right), \]

where \( V^r_{T+1}(W_1, j) \) and \( V^s_{T+1}(W_1, OP, j) \) are again defined as in Equation (12) and (9), respectively, and \( \hat{V}_{T+\tau} = 0, \forall \tau > \omega - x. \)

Note that the new value function when retaining, \( \hat{V}_T(W_0, j) \), will generally increase since it now also incorporates the possibility of settling in future periods. Therefore, when evaluating the benefit of settling at time \( T \), the policyholder now will also take into account the opportunity cost of not being able to settle later on. This will obviously lessen the willingness of the policyholder to settle in the current period, and will thus truncate the threshold set \( \Omega(\bar{z}) \). Hence, the effect of adverse selection on pricing will be even more pronounced in this case.

Furthermore, note that the value function \( \hat{V}_T \) now is nonlinear in the future survival probabilities \( \tau p_{x+1}(T+1) \). Thus, while in the case of one settlement date as in the previous subsections only concurrent life expectancies in the form of the concurrent generation life table matter, when settling is possible in multiple periods, systematic mortality risk at the population level becomes material.

4 Application

In this section, we evaluate exemplary life settlement transactions under asymmetric information to investigate the effect of adverse selection on the secondary life market. More precisely, we first compile population generation life tables based on projections of historical mortality data as well as individual life tables from these population life tables and the Gamma frailty model. Then, we introduce our assumptions on the policyholder’s preferences to derive offer prices as outlined in the previous section. All of our calculations are based on a representative female policyholder who
purchased her whole-life policy in year $t = 1978$ at age 50 and who in year $T = 2008$ is looking to settle the policy at age $x = 80$. For simplicity, we only consider the case of a single possible settlement date.

### 4.1 Projections of Population/Individual Generation Life Tables

Generation life tables for the female population are derived using the Lee and Carter (1992) methodology\(^8\) based on U.S. mortality data as available from the *Human Mortality Database*.\(^9\) More precisely, we generate two series of forecasts: First, in 1978, the life insurer uses historical data from 1958 to 1977 to generate forecasts of future survival probabilities when setting the fair level annual premium amount. Then, in 2008, the life settlement company additionally incorporates mortality experience from 1978 to 2007 to update the generation life table for pricing the life settlement transaction. Figure 1(a) and 1(b) display the projected population survival probabilities for an – in year 1978 – 50-year old female and an – in year 2008 – 80-year old female, respectively. Note that although we consider the same individual at two different points in time, the probability of surviving until some particular age will change since – at age 80 – we observe *conditional* survival probabilities taking into account that the individual has not deceased between the ages of 50 and 80. For instance, while the probability of surviving until the age of 90 is significantly less than 40% for the 50-year old in 1978, it is close to 50% for the 80-year old in 2008. But even when accounting for this effect, under our specification, *conditional* survival probabilities are also adjusted due to updates in the mortality forecast based on the mortality experience between 1978 and 2007.

In deriving the annual premium in year 1978, for simplicity we disregard expenses and profit margins, and assume a constant risk-free rate $R^f = 4\%$. The equivalence principle then yields a level annual premium of $16.25$ per $1,000$ death benefit.

---

\(^8\)Here $\{\alpha_t\}$ and $\{\beta_t\}$ in the Lee-Carter model are estimated via the weighted least-squares algorithm, and $\{\kappa_t\}$ is further adjusted by fitting a Poisson regression model (cf. Booth et al. (2002)).

\(^9\)Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de.
Figure 1: Population survival probabilities projected from the Lee-Carter method for the same U.S. female cohort when aged 50 in year 1978 (left panel) and when aged 80 in year 2008 (right panel).

To generate heterogeneous lifetime distributions from Equations (6)/(7) and the compiled population generation life table in year 2008, we set the square of the coefficient of variation \( \gamma^2 = 0.288 \) as estimated in Manton et al. (1986) based on cohort-specific Medicare data and a Gompertz mortality model. Moreover, we choose the mean frailty \( \zeta = 1.187 \). This choice ascertains that the (unconditional) actuarially fair offer price is unbiased, that is the unconditional \( OP^a \) is given by Equation (10) and population mortalities \( z_j^1 = 1 \). To illustrate these assumptions, Figure 2 shows the empirical cumulative distribution functions for \( \tau p_{80}^j(2008) \) based on 1,000,000 simulations with \( \tau \) at 1, 11, and 21, respectively. We observe that while the medians of the individualized survival probabilities are roughly in line with the corresponding expected values from panel (b) of Figure 1, the survival probabilities display quite a bit of dispersion. For instance, for the “healthiest” 10% of the cohort of 80 year old females in 2008, the probability of surviving until age 91 is significantly above 65%, whereas for the 10% of the cohort with the most impaired health states, the probability of surviving until age 91 is below 20%.
Figure 2: Empirical cumulative distribution functions of age-80 heterogeneous individual survival probabilities in year 2008 with terms $\tau = 1$, 11, and 21, based on the Gamma frailty model with $\gamma^2 = 0.288$ and $\zeta = 1.187$. 

(a) $\mathbb{P}_{80}^{2008}(j)$

(b) $\mathbb{P}_{80}^{2008}(j)$

(c) $\mathbb{P}_{80}^{2008}(j)$
4.2 Settlement Decision and Offer Prices: The Base Case

In our numerical application, for the utility function $u(\cdot)$, we use a standard constant relative risk aversion (CRRA) form: $u(c) = \frac{c^{1-\rho}}{1-\rho}$. The bequest function, $v(\cdot)$, is assumed to be of the form

$$v(W) = \frac{1 + R^f}{R^f} \times \frac{(\frac{R^f}{1 + R^f} W)^{1-\rho}}{1 - \rho}.$$ 

This assumption entails that the bequest amount $W$ is transferred into a perpetuity with periodic payment $\frac{R^f}{1 + R^f} W$ to the beneficiary, and thus we define the bequest function as the sum of the series of corresponding utility functions with the time discount factor $\beta = \frac{1}{1 + R^f}$. The relative risk-aversion parameter $\rho$ is chosen at 1.584 as calibrated in Hall and Jones (2007). Moreover, we assume the initial wealth of the policyholder at the beginning of year 2008, $W_0$, is $500,000$, whereas the death benefit of her whole-life policy, $F$, is $1,000,000$. From Section 4.1, the annual contingent premium is then calculated as $16,245$. Finally, with the risk-free interest rate fixed at 4%, the time discount factor $\beta$ is set at $\frac{1}{1.04}$.

The offer price under symmetric information, $OP^{sym}$, can be directly obtained from Equation (10). For simplicity, we assume that the estimated expected lifetime distribution is given by the population life table, i.e. $\bar{z} = 1$, and that the estimation function $f(z^j | \bar{z})$ is identical to $f(z^j)$, i.e. there is no prior information available.\(^{10}\)

By comparing the optimal value functions from Equations (8) and (9), we can calculate the associated reservation price for each policyholder type, i.e. the minimum offer price so that the policyholder of type $z^j$ will accept the settlement offer. The result is shown in Figure 3(a). In addition, Figure 3(a) displays $OP^{sym}$ for $z^j = \bar{z} = 1$ with hurdle rates $R$ at 4%, 8%, and 12%. From the figure, we observe that at each hurdle rate, the associated $OP^{sym}$ crosses with the reservation price curve.\(^{11}\) Hence, only policyholders with relatively better private lifetime estimates

\(^{10}\)This assumption is justified if the generation life table in our case is interpreted as the one supplied by the life expectancy provider.

\(^{11}\)From Figure 3(a), we also observe that the theoretical reservation price will become negative when the frailty factor is close to 0. This is not surprising since for extremely healthy policyholders (very small $z^j$), the burden of paying future contingent insurance premiums exceeds the limited gain from the death benefit. Of course, in these cases, policyholders can simply lapse their policies.
Figure 3: Asymmetric behavior of policyholders. The left panel shows policyholders’ reservation prices under different frailty factors $z^j$, as well as the actuarially fair offer prices under three hurdle rate assumptions. The right panel shows the threshold $z^*$ for varying offer prices. Only policyholders with $z^j$ less than $z^*$ will choose settling over retaining.

will actually choose to settle their policies at $OP_{sym}$, whereas the rest will choose to retain their policies. In particular, the effect of adverse selection increases with $R$. Figure 3(b) now illustrates the threshold set $\Omega(OP) = [0, z^*(OP)]$ by plotting $z^*$ for $OP$ varying from $0$ to $700,000$.

For given $z^*(OP)$, we continue to calculate the equilibrium offer price $OP_e$ for our representative policyholder via Equation (11). In the base case, we set the hurdle rate $R$ equal to the risk free rate $R^f$ (4%). Figure 4(a) shows the expected profit for the life settlement company as a function of the offer price. We find that the expected profit first increases with the offer price, then it decreases until it eventually becomes negative. The equilibrium offer price $OP_e = 524,030$ is identified as the point where the expected profit curve crosses the zero profit line. In particular, it is smaller than $OP^a = 544,260$ based on $R = 4\%$, which implies a modest effect of adverse selection. For illustrative purposes, we determine the interest rate $R'$ resulting in the equilibrium offer price when using the actuarially fair pricing formula for $OP^a$. We obtain $R' = 4.39\%$. Thus, by setting an ex-ante hurdle rate of 4.39% and using the actuarially fair pricing formula without the consideration of asymmetric information, the ex-post profit rate for the life settlement company

---

12In particular, our calculations show that for given $OP$, only policyholders with $z^j$ less than a threshold $z^*(OP)$ will settle, i.e. $\Omega(OP) = [0, z^*(OP)]$, which numerically verifies the assumption in Proposition 3.1.
Figure 4: Panel (a): Expected profits and equilibrium offer price under asymmetric information with $R = 4\%$. We observe that the expect profit first increases then decreases with the offer price, with an equilibrium offer price $OP^e = $524,030 identified as the zero expected profit point. $OP^a = $544,260 denotes the actuarially fair price based on $R = 4\%$. Panel (b): Optimal first period consumptions under retaining and settling. We observe that the optimal consumption net of premium when retaining increases with $z^j$ but is always smaller than when settling.

will be the risk free rate $R^f = 4\%$. The discrepancy of “expected” and “realized” rate in the base case is thus 0.39\%.

Figure 4(b) shows how the policyholder’s first period optimal consumption, $c_0$, differs with a varying frailty factor $z^j$ under retaining and settling. We find that the optimal consumption when retaining increases considerably: Individuals with impaired health states consume more because they are less likely to require funds for consumption in the far future. Relative to this stark variation, the corresponding pattern when settling appears essentially flat – although our numerical results show that optimal first-period consumption also mildly increases in this case. We also notice that actual consumption net of premium expenditure, $c_0 - P$, when retaining will always be smaller than the optimal consumption in the case of settling in the considered range. This appears natural at least for those policyholders who prefer settling to surrendering, i.e. those with $z^j \leq z^*(OP^e) = 1.926$, since they are happy to trade off the death benefit associated with retaining the policy for an increase in current consumption.
4.3 Sensitivity Analyses

In what follows, we demonstrate how the results change when varying the assumptions in the base case. In particular, we consider changes in the life settlement company’s target hurdle rate, the policyholder’s wealth/insurance ratio, the risk aversion parameter, and the bequest motive.

Hurdle Rate

We first calculate the offer price under symmetric information, $OP^{sym}$, for different choices of the hurdle rate $R$ based on Equation (10). Similarly as in the base case, here we assume $z_j = z = 1$. Contingent on the obtained $OP^{sym}$, the policyholder makes her optimal decision by comparing the value functions under retaining and settling. Figure 5 shows $OP^{sym}$ as well as the associated value functions $V^{r}_{T}$ and $V^{s}_{T}$ for hurdle rates $R$ between 4% and 12%. In particular, we observe from Panel (b) that the secondary market transaction will only be executed when $R \leq 9.95\%$ or, equivalently, $OP^{sym} \geq \$323,370$.

Table 1 displays the equilibrium offer prices ($OP^{e}$), the actuarially fair offer prices ($OP^{a}$), and the equivalent ex-ante rates ($R^{e}$) as explained in the previous subsection for hurdle rates varying from 4% to 8% and other assumptions as in the base case. From the table, it is clear that while in all cases adverse selection is material as indicated by the differences between $OP^{e}$ and $OP^{a}$, the effect becomes more pronounced as $R$ increases. More specifically, while in the base case the discrepancy between the ex-ante actuarially fair rate $R^{e}$ and the equivalent ex-post rate $R$ is a mere 0.39%, it can be as high as 5.72% for $R = 8\%$. This is in line with the observations from Figure 3(a), which shows that the larger the hurdle rate, the smaller the offer price, and thus the smaller the portion of policyholders who prefer settling to retaining – which eventually governs the effect of adverse selection.

Wealth/Insurance Ratio

To analyze the sensitivity of the ratio between policyholder wealth and the face value of the insurance policy, we fix the face value $F$ at $1,000,000$ and vary the initial wealth $W_0$ from $300,000$.
Figure 5: Offer prices under symmetric information. The left panel shows the offer prices under different hurdle rates. The right panel shows the value functions when retaining or settling under different hurdle rates, which indicates that the policyholder will only settle when $R \leq 9.95\%$.

<table>
<thead>
<tr>
<th>Ex-Post Hurdle Rate: $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
</tr>
<tr>
<td>$OP^e$</td>
</tr>
<tr>
<td>$OP^a$</td>
</tr>
<tr>
<td>$R'$</td>
</tr>
</tbody>
</table>

Table 1: Sensitivity tests on the ex-post hurdle rates ($R$) varying from 4% to 8%. Here, $OP^e$ denotes the equilibrium offer price under the given hurdle rate, $OP^a$ denotes the corresponding actuarially fair price based on population-weighted mortalities, and $R'$ is the ex-ante actuarially fair rate that gives the hurdle rate when considering asymmetric information. We observe a stronger effect of adverse selection as $R$ increases.
Table 2: Sensitivity tests on the wealth/insurance ratio ($W_0/F$) varying from 0.3 to 0.7. Here, $OP^e$ denotes the equilibrium offer price, $OP^a$ denotes the corresponding actuarially fair price based on population-weighted mortalities, and $R'$ is the ex-ante actuarially fair rate that yields the hurdle rate $R = 4\%$ when considering asymmetric information. We observe a stronger effect of adverse selection as $W_0/F$ increases.

<table>
<thead>
<tr>
<th>Wealth/Insurance Ratio: $W_0/F$</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$OP^e$</td>
<td>$543,350$</td>
<td>$539,670$</td>
<td><strong>$524,030$</strong></td>
<td>$498,940$</td>
<td>$461,990$</td>
</tr>
<tr>
<td>$OP^a$</td>
<td>$544,260$</td>
<td>$544,260$</td>
<td><strong>$544,260$</strong></td>
<td>$544,260$</td>
<td>$544,260$</td>
</tr>
<tr>
<td>$R'$</td>
<td>4.02%</td>
<td>4.09%</td>
<td><strong>4.39%</strong></td>
<td>4.91%</td>
<td>5.74%</td>
</tr>
</tbody>
</table>

Table 2: Sensitivity tests on the wealth/insurance ratio ($W_0/F$) varying from 0.3 to 0.7. Here, $OP^e$ denotes the equilibrium offer price, $OP^a$ denotes the corresponding actuarially fair price based on population-weighted mortalities, and $R'$ is the ex-ante actuarially fair rate that yields the hurdle rate $R = 4\%$ when considering asymmetric information. We observe a stronger effect of adverse selection as $W_0/F$ increases.

to $700,000$, i.e. we vary the wealth/insurance ratio $W_0/F$ from 0.3 to 0.7. We focus on varying the ratio since under the assumed CRRA utility/bequest functions, proportional changes are immaterial to optimal choices. The associated results are summarized in Table 2 with all other assumptions as in the base case.

We observe that the effect of adverse selection increases with the wealth/insurance ratio: With higher initial wealth $W_0$, the policyholder faces less pressure from the fixed periodic expenditure of the insurance premium $P$, and thus has less incentive to settle her policy. Therefore, for any given offer price $OP$, the threshold $z^*(OP)$ is reduced and the effect of adverse selection increases. For example, when $W_0/F = 0.7$, an ex-ante hurdle rate of 5.74\% is required to yield an ex-post rate of 4\% when considering asymmetric information, whereas for $W_0/F = 0.3$, the equilibrium offer price is very close to the actuarially fair price and the difference between ex-ante and ex-post rate is merely 0.02\%.

**Risk Aversion Parameter**

Estimates for the value (or range) of an individual’s relative risk aversion vary considerably throughout the literature. While some estimate a relative risk aversion coefficient close to one (see e.g. Chetty (2006)), others find empirical support for considerably higher values (see e.g. Cohen and
Table 3: Sensitivity tests on the risk aversion parameter (\( \rho \)) varying from 0.8 to 2.5. Here, \( OP^e \) denotes the equilibrium offer price, \( OP^a \) denotes the corresponding actuarially fair price based on population-weighted mortalities, and \( R' \) is the ex-ante actuarially fair rate that yields the hurdle rate \( R = 4\% \) when considering asymmetric information. We observe a weaker effect of adverse selection as \( \rho \) increases.

Bequest Motive

The last test we conduct is with respect to the policyholder’s bequest motive, \( b \). The bequest motive measures how much the beneficiaries’ utility translates into the policyholder’s utility function. Therefore, the bequest function, in the general form, can be defined as

\[
v(W) = b \times \frac{1 + R^f}{R^f} \times \frac{\left(\frac{R^f}{1 + R^f} W\right)^{1-\rho}}{1 - \rho} = b' \times \frac{W^{1-\rho}}{1 - \rho}
\]

for \( b' = b \times \left(\frac{R^f}{1 + R^f}\right)^{-\rho} \) and we choose \( b = 1 \) in the base case. Table 4 displays the results with \( b \) varying from 0.1 to 1.0 and all other assumptions as in the base case.
We find that the effect of adverse selection increases with the bequest motive. This is natural, since the higher the bequest motive, the more valuable the life insurance policy is to the policyholder, and the more likely she is to keep the policy. In particular, with the bequest motive as low as 0.1, i.e. when the utility of the life policy is small, the equilibrium offer price coincides with the actuarially fair price. Hence, in this case we observe no effect of adverse selection since all policyholders will be willing to settle their policies.

### 4.4 Empirical Implications and Limitations

The numerical analyses in the previous subsections demonstrate that asymmetric information can have a significant effect on pricing. To illustrate, we calculate the ex-ante rate $R'$ based on the supplied mortality rates that is required to give a certain ex-post rate $R$ when taking private information into account, and we find that the difference between the two can be considerable. Thus, when performing expected return calculations without taking this effect into account – i.e. when basing them on conventional actuarial pricing formulas as it is common within the industry – it is natural to expect a discrepancy between these “expected” and realized returns, i.e. information asymmetry may dampen the perceived profitability of a life settlement fund.

The size of the difference between ex-ante and ex-post rates depends on various parameters

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**Table 4:** Sensitivity tests on the bequest motive ($b$) varying from 0.1 to 1.0. Here, $O^p_e$ denotes the equilibrium offer price, $O^p_a$ denotes the corresponding actuarially fair price based on population-weighted mortalities, and $R'$ is the ex-ante actuarially fair rate that yields the hurdle rate $R = 4\%$ when considering asymmetric information. We observe a stronger effect of adverse selection as $b$ increases.

<table>
<thead>
<tr>
<th>Bequest Motive: $b$</th>
<th>0.1</th>
<th>0.35</th>
<th>0.55</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O^p_e$</td>
<td>$544,260$</td>
<td>$543,350$</td>
<td>$541,450$</td>
<td>$533,370$</td>
<td><strong>$524,030$</strong></td>
</tr>
<tr>
<td>$O^p_a$</td>
<td>$544,260$</td>
<td>$544,260$</td>
<td>$544,260$</td>
<td>$544,260$</td>
<td><strong>$544,260$</strong></td>
</tr>
<tr>
<td>$R'$</td>
<td>4%</td>
<td>4.02%</td>
<td>4.05%</td>
<td>4.21%</td>
<td><strong>4.39%</strong></td>
</tr>
</tbody>
</table>
such as the target hurdle rate, policyholder wealth, risk aversion level etc., but the levels we find could potentially explain an ample part of the discrepancies reported in the literature. For instance, Gatzert (2010) reports expected returns calculated on a policy-by-policy basis between 8% and 12% p.a., whereas Braun et al. (2012) find that open-end life settlement funds between 2004 and 2010 on average returned approximately 4.8% p.a. – resulting in a discrepancy of roughly 3-7% p.a. The difference between the ex-ante and ex-post rates, $R' - R$, in our example calculations, on the other hand, ranges from zero to almost 6%, although it is possible that generalizations of the model framework – e.g. the possibility to settle in future periods as indicated in Section 3.4 – will yield even more pronounced effects of asymmetric information.

However, it is important to note that of course the assumptions here are not necessarily applicable to actual life settlement transactions, and our framework ignores several factors that may affect the decision to settle a policy. For instance, we use mortality and frailty estimates based on population data rather than data from life expectancy providers. On the other hand, we focus on time-separable preference specifications where marginal utilities are independent of the health state, which e.g. prevents settling due to liquidity constraints associated with increasing healthcare costs. Thus, more research is necessary to ascertain if and how much of the discrepancy between expected and realized returns in the life settlement market can be explained by our results – although preliminary analyses indicate that asymmetric information is indeed relevant in the life settlement market (see Ruß and Bauer (2012)).

5 Conclusion

In this paper, we discuss the effect of asymmetric information with respect to policyholders’ lifetime distributions on the pricing of life settlements in a competitive equilibrium. The key insight is that the possibility for the policyholder to walk away from “bad” offers while gladly agreeing to “good” offers will result in a shifted pricing schedule. In particular, the resulting equilibrium offer price will be lower than the “actuarially fair” price calculated on the basis of best estimate survival
probabilities and a hurdle rate set according to the investors’ return expectations – which is the benchmark used in practice. Thus, information asymmetries may explain the discrepancy between “expected” and realized returns within the life settlements market, an observation that previously was simply attributed to “errors” in life expectancy estimates.

We derive coherent pricing formulas that account for this effect in a simple lifetime utility framework, and we conduct example calculations. Our results suggest that information asymmetries can have a crucial impact on pricing secondary life insurance market transactions, depending on the underlying model specification and the parameter assumptions – and could potentially explain an ample part of the discrepancies in the market.

Future research is necessary to quantify these effects based on data from the actual life settlement market. Nonetheless, our considerations provide a new angle on the financial analysis of life settlements, and therefore shed light on the nature of the “unique risks” within this market as recently discussed in the financial press.

References


Appendix

A Proof of Proposition 2.1

Denote by $G(p|\bar{p})$ the cumulative distribution function of $f(p|\bar{p})$. In order to prove $OP^c(\bar{p}) \leq OP^a(\bar{p})$, it is sufficient to show that

$$0 \geq \int_{p^*(OP^a(\bar{p}))}^{1} ((1-p) \times F - OP^a(\bar{p}) \times (1 + R^f)) f(p|\bar{p}, p \geq p^*(OP^a(\bar{p}))) \, dp$$

$$\Leftrightarrow 0 \geq \frac{1}{1 - G(p^*(OP^a(\bar{p})))} \times \int_{p^*(OP^a(\bar{p}))}^{1} ((1-p) \times F - OP^a(\bar{p}) \times (1 + R^f)) f(p|\bar{p}) \, dp$$

$$\Leftrightarrow 0 \geq F \times \int_{p^*(OP^a(\bar{p}))}^{1} ((1-p) - E[(1-p)|\bar{p}]) f(p|\bar{p}) \, dp$$

$$\Leftrightarrow 0 \geq F \times \int_{p^*(OP^a(\bar{p}))}^{1} f(p|\bar{p}) \, dp \times (E[(1-p)|\bar{p}, p \geq p^*(OP^a(\bar{p}))] - E[(1-p)|\bar{p}])$$

$$\Leftrightarrow 0 \leq E[p|\bar{p}, p \geq p^*(OP^a(\bar{p}))] - E[p|\bar{p}],$$

which is trivially satisfied.

\[\blacksquare\]

B Proof of Proposition 3.1

We start by showing that $OP^{sym}(j)$ is increasing in $z^j$. With Equation (7) and (10), we obtain:

$$\frac{\partial}{\partial z^j} OP^{sym} = \sum_{\tau=1}^{\omega-x} \left( - \int_0^{\tau-1} \mu_T(s, x) \, ds \right) \exp \left\{ - \int_0^{\tau-1} \mu_T(s, x) \, ds \times z^j \right\} \left[ F \frac{\left(1+R\right)^\tau - P}{(1+R)^{\tau-1}} \right]$$

$$- \left( - \int_0^{\tau} \mu_T(s, x) \, ds \right) \exp \left\{ - \int_0^{\tau} \mu_T(s, x) \, ds \times z^j \right\} \frac{F}{(1+R)^\tau}$$

$$= \sum_{\tau=1}^{\omega-x} \left( \int_0^{\tau-1} \mu_T(s, x) \, ds \right) \exp \left\{ - \int_0^{\tau-1} \mu_T(s, x) \, ds \times z^j \right\} \frac{F}{(1+R)^{\tau-1}}$$

$$+ \sum_{\tau=1}^{\omega-x} \left( \int_0^{\tau} \mu_T(s, x) \, ds \right) \exp \left\{ - \int_0^{\tau} \mu_T(s, x) \, ds \times z^j \right\} \frac{F}{(1+R)^{\tau-1}}$$

$$- \sum_{\tau=1}^{\omega-x} \left( \int_0^{\tau-1} \mu_T(s, x) \, ds \right) \exp \left\{ - \int_0^{\tau-1} \mu_T(s, x) \, ds \times z^j \right\} \frac{F}{(1+R)^{\tau-1}}$$
\[
+ \sum_{\tau=1}^{\omega-x} \left( \int_0^{\tau-1} \mu_T(s, x) \, ds \right) \exp \left\{ - \int_0^{\tau-1} \mu_T(s, x) \, ds \times z^{j} \right\} \left[ \frac{F}{(1 + R)^{\tau-1}} - \frac{F}{(1 + R)^{\tau}} \right] \\
= \sum_{\tau=1}^{\omega-x} \left( \int_0^{\tau-1} \mu_T(s, x) \, ds \right) \exp \left\{ - \int_0^{\tau-1} \mu_T(s, x) \, ds \times z^{j} \right\} \left[ \frac{P}{(1 + R)^{\tau-1}} + \frac{RF}{(1 + R)^{\tau}} \right] \\
\geq 0.
\]

In order to prove \( OP^x(\bar{z}) \leq OP^a(\bar{z}) \), it is now sufficient to show that

\[
\int_{\Omega(OP^a(\bar{z}))} \left( \sum_{\tau=1}^{\omega-x} \left[ (\tau-1)p^j_x(T) - \tau p^j_x(T) \right] \times \frac{F}{(1 + R)^{\tau}} \right. \\
- \left. \tau-1p^j_x(T) \times \frac{P}{(1 + R)^{\tau-1}} \right) - OP^a(\bar{z}) \right) f \left( z^j | \bar{z}, z^j \in \Omega(OP^a(\bar{z})) \right) \, d z^j < 0
\]

\( \Leftrightarrow \)

\[
\int_0^{z^*(OP)} (OP^{sym}(j) - OP^a(\bar{z})) f \left( z^j | \bar{z}, z^j \leq z^*(OP) \right) \, d z^j < 0
\]

\( \Leftrightarrow \)

\[
\int_0^{z^*(OP)} (OP^{sym}(j) - OP^a(\bar{z})) f \left( z^j | \bar{z} \right) \, d z^j < 0
\]

\( \Leftrightarrow \)

\[
\int_0^{z^*(OP)} f(z^j | \bar{z}) \, d z^j \times (E^j[OP^{sym}(j) | \bar{z}, 0 \leq z^j \leq z^*(OP)] - E^j[OP^{sym}(j) | \bar{z}]) < 0,
\]

which is trivially satisfied given the monotonicity of \( OP^{sym}(j) \).